Parity, Charge Conjugation and $SU(3)$ Constraints on Threshold Enhancement in J/ψ decays into $\gamma p\bar{p}$ and $Kp\bar{\Lambda}$

Xiao-Gang He* and Xue-Qian Li†

Department of Physics,

Nankai University, Tianjin, China

J.P. Ma‡

Institute of Theoretical Physics,

Academia Sinica, Beijing, China

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Abstract

We study the threshold enhancement effects of baryon-anti-baryon systems in J/ψ decays at BES using parity P, charge conjugation C, and flavor $SU(3)$ symmetries. The P and C symmetries restrict the $p\bar{p}$ in $J/\psi \to \gamma p\bar{p}$ to be in a state with $C = +1$, while the $p\bar{p}$ in $J/\psi \to \pi^0 p\bar{p}$ to be with $C = -1$. Combining the C and P symmetries with flavor $SU(3)$ symmetry, i.e. the CPS symmetry, we find that the $\Lambda\bar{p}$ system cannot be in 0^+ and 0^- states in $J/\psi \to K^+\Lambda\bar{p}$. We provide a consistent explanation of observation and non-observation of near threshold enhancements in $J/\psi \to K^+\Lambda\bar{p}$ and $J/\psi \to \pi^0 p\bar{p}$, respectively. We also find that near baryon pair threshold enhancement can happen in several channels for J/ψ decays and can be several times larger than that observed in $J/\psi \to K^+\Lambda\bar{p}$ in some channels.

*Also at Department of Physics, Peking University; On leave of absence from Taiwan University, Taipei; Electronic address: hexg@phys.ntu.edu.tw

†Electronic address: lixq@nankai.edu.cn

‡Electronic address: majp@itp.ac.cn

Recently the BES collaboration has observed enhancements in $J/\psi \to \gamma p\bar{p}$[1] and $J/\psi \to K p\bar{\Lambda}$[2] with the invariant masses of the $p\bar{p}$ and $p\bar{\Lambda}$ near their thresholds, whereas no enhancement in $J/\psi \to \pi^0 p\bar{p}$[1] is seen. Similar enhancement effects have also been observed in $B^+ \to K^+ p\bar{p}$ and $\bar{B}^0 \to D^0 p\bar{p}$[3]. There are no known states corresponding to the required resonant masses. These events are quite anomalous.

These anomalous events have stimulated a number of theoretical speculations[4, 5, 6, 7, 8, 9, 10, 11]. In [4] the enhancement is explained by an ${}^1S_0$ bound state of $p\bar{p}$. The binding energy and lifetime of such a state and also a P-wave state is studied within the linear σ model[5]. In [6] scalar glueball mechanism is invoked. Several other mechanisms are also discussed. The effect of final state interactions through one pion exchange is studied in [7] and it is found that the enhancement can be partly reproduced. In [8] LEAR data are used to fix scattering lengthes of $p\bar{p}$ scattering near the threshold and with these fixed scattering lengthes an enhancement near the threshold of $J/\psi \to \gamma p\bar{p}$ can be produced and also an explanation of no enhancement in $J/\psi \to \pi^0 p\bar{p}$ can be given. In [9] a 0^{-+} baryonium close to the threshold is predicted with a Skyrmion-type potential. In [10] it is suggested that the enhancement in $J/\psi \to \gamma p\bar{p}$ may be fitted as a cusp. All of these works can more or less explain the enhancement in $\gamma p\bar{p}$, but a decisive explanation is still needed. In [11] possible quantum numbers of the possible resonant state in $\gamma p\bar{p}$ and its possible decay modes are discussed.

Although the dynamical mechanism responsible for these anomalous events is not understood in detail, the strong and/or electromagnetic interaction should be responsible for the decays and the enhancement near the threshold. The decays must respect parity P and charge conjugation C symmetries. These symmetries can provide important information about the enhancement effects. We will perform such an analysis for the decays of a J/ψ into $\gamma p\bar{p}$, $\pi^0 p\bar{p}$. We further carry out a more general analysis for the decays of a J/ψ into $\gamma B'\bar{B}$, $MB'\bar{B}$, where M stands for an octet meson in the flavor $SU(3)$ symmetry, and $B'(\bar{B})$ stands for an octet baryon(anti-baryon) trying to use $SU(3)$ symmetry to relate and to constrain $\pi^0 p\bar{p}$ and $K^+\Lambda\bar{p}$ and provide new predictions for processes involving other baryons and mesons, i.e., $J/\psi \to \gamma B'\bar{B}$ and $J/\psi \to MB'\bar{B}$.

We first consider possible constraints on decay amplitudes of J/ψ from P and C symmetries. We take $J/\psi \to \gamma p\bar{p}$ as an example for some details of our analysis. For convenience of identifying quantum numbers of the baryon pairs near their thresholds, we study the decay

amplitude in the rest-frame of $p\bar{p}$. We have

$$J/\psi(\vec{p},\vec{\varepsilon}_J) \to \gamma(\vec{p},\vec{\varepsilon}) + p(\vec{q}) + \bar{p}(-\vec{q}), \tag{1}$$

where $\vec{\varepsilon}_J$ and $\vec{\varepsilon}$ are 3-vectors for the polarizations of the J/ψ and photon, respectively. The particle 3-momenta are indicated in brackets. The decay amplitude can be written with standard Dirac spinors as:

$$\mathcal{T}(J/\psi \to \gamma p\bar{p}) = \bar{u}(\vec{q})A(\vec{\varepsilon}_J,\vec{\varepsilon},\vec{p},\vec{q})v(-\vec{q}), \tag{2}$$

with A being a 4×4 matrix. For our purpose it is convenient to work with two-component spinors using $\psi^\dagger$ for the proton and χ for the anti-proton.

The Dirac spinor $\bar{u}(\vec{q})(v(-\vec{q}))$ can be expressed with $\psi^\dagger$ (χ), $\vec{q}$ and $\vec{\sigma}$, where $\sigma^i(i=1,2,3)$ are Pauli matrices. The decay amplitude can be then written with these two-component spinors as:

$$\mathcal{T}(J/\psi \to \gamma p\bar{p}) = \psi^\dagger\left(F_{\gamma 0}(\vec{p},\vec{q}) + \vec{\sigma}\cdot\vec{F}_{\gamma\sigma}(\vec{p},\vec{q})\right)\chi. \tag{3}$$

The form factors $F_{\gamma 0}(\vec{p},\vec{q})$ and $\vec{F}_{\gamma\sigma}(\vec{p},\vec{q})$ also depend on polarization vectors. Near the threshold of the $p\bar{p}$ system, one can expand the form factors in $|\vec{q}|$. In the expansion one identifies the $p\bar{p}$ system with quantum numbers according to the total spin S, the orbital angular momentum L and the total angular momentum $J=L+S$, in short ${}^{2S+1}L_J$. The P and C eigenvalues of the system are given by $(-1)^{L+1}$ and $(-1)^{L+S}$, respectively.

The form factor $F_{\gamma 0}$ can be written as:

$$F_{\gamma 0} = \varepsilon_J^i\varepsilon^j T_0^{ij}(\vec{p},\vec{q}), \tag{4}$$

while the tensor T_0^{ij} can be decomposed with the following 9 tensors of rank 2 because of rotation covariance: $\delta_{ij},\ \varepsilon_{ijk}p^k,\ \varepsilon_{ijk}q^k,\ \varepsilon_{ijk}n^k, p^{\{i}p^{j\}},\ q^{\{i}q^{j\}},\ p^{\{i}q^{j\}},\ p^{\{i}n^{j\}},\ n^{\{i}q^{j\}}$, with $\vec{n}=\vec{p}\times\vec{q}$. The notation $S^{\{ij\}}$ means that S is symmetric and trace-less. Parity conservation eliminates terms proportional to $\delta_{ij},\ \varepsilon_{ijk}n^k, p^{\{i}p^{j\}},\ q^{\{i}q^{j\}},\ p^{\{i}q^{j\}}$ in T_0^{ij}. Hence the tensor is written as:

$$\begin{aligned} T_0^{ij}(\vec{p},\vec{q}) &= \varepsilon_{ijk}p^k b_1(m_{p\bar{p}},\nu) + \varepsilon_{ijk}q^k b_2(m_{p\bar{p}},\nu) \\ &+ p^{\{i}n^{j\}}b_3(m_{p\bar{p}},\nu) + q^{\{i}n^{j\}}b_4(m_{p\bar{p}},\nu), \end{aligned} \tag{5}$$

with $\nu=\vec{p}\cdot\vec{q}$. It should be noted that $b_i(i=1,2,3,4)$ are Lorentz invariant form factors, they depend on the invariant mass $m_{p\bar{p}}$ of the $p\bar{p}$ system and ν, the later can be expressed

with $m_{p\bar{p}}$ and the invariant mass of the $p\gamma$ system. The symmetry of charge conjugation gives constraints of these form factors: $b_1(m_{p\bar{p}},\nu) = b_1(m_{p\bar{p}},-\nu)$, $b_2(m_{p\bar{p}},\nu) = -b_2(m_{p\bar{p}},-\nu)$, $b_3(m_{p\bar{p}},\nu) = -b_3(m_{p\bar{p}},-\nu)$, $b_4(m_{p\bar{p}},\nu) = b_4(m_{p\bar{p}},-\nu)$. Near the threshold, i.e., $\vec{q}\to 0$, one can expand these form factors in ν. This expansion is equivalent to the standard partial wave analysis. Throughout this work we will only consider contributions up to D-wave states because contributions from states with $L > 2$ are suppressed by at least $|\vec{q}|^3$ which are small compared with baryon masses near the threshold. With this approximation $F_{\gamma 0}$ takes the form:

$$
\begin{aligned}
& F_{\gamma 0} = (\vec{\varepsilon}\times\vec{\varepsilon}_J)\cdot\vec{p}\mathcal{S}_\gamma + \left(q^i q^j - \frac{1}{3}|\vec{q}|^2\delta_{ij}\right) \\
& \cdot\Big[(\vec{\varepsilon}\times\vec{\varepsilon}_J)\cdot\vec{p}p^ip^j\mathcal{D}_{1\gamma} + (\vec{\varepsilon}\times\vec{\varepsilon}_J)^i p^j\mathcal{D}_{2\gamma} \\
& +\vec{\varepsilon}_J\cdot\vec{p}(\vec{\varepsilon}\times\vec{p})^i p^j\mathcal{D}_{3\gamma} + (\varepsilon^i(\vec{\varepsilon}_J\times\vec{p})^j \\
& ++\varepsilon_J^i(\vec{\varepsilon}\times\vec{p})^j)\mathcal{D}_{4\gamma}\Big] + \cdots,
\end{aligned} \tag{6}
$$

where we have used $\vec{p}\cdot\vec{\varepsilon} = 0$ for the photon. All the form factors $\mathcal{S}_\gamma$, $\mathcal{D}_{i\gamma}(i = 1,2,3,4)$ only depend on $m_{p\bar{p}}$.

One can also make a general decomposition for $\vec{F}_{\gamma\sigma}$. It has the form:

$$
F^i_{\gamma\sigma} = \varepsilon^j_J\varepsilon^k T^{ijk}_\sigma(\vec{p},\vec{q}). \tag{7}
$$

The constraints for the tensors are: $T^{ijk}_\sigma(\vec{p},\vec{q}) = -T^{ijk}_\sigma(-\vec{p},-\vec{q})$ from parity, and $T^{ijk}_\sigma(\vec{p},\vec{q}) = -T^{ijk}_\sigma(\vec{p},-\vec{q})$ from charge conjugation. From the above analysis one can easily see that $\vec{F}_{\gamma\sigma}$ represents the contributions of the states with $L = 1,3,\cdots$, i.e., P-wave states, F-wave states, $\cdots$. The general expression is complicated since the tensor T^{ijk} is of rank 3. Using the fact $\vec{\varepsilon}\cdot\vec{p} = 0$, we have for the leading non-zero terms with $l = 1$:

$$
T^{ijk}_\sigma = \vec{p}\cdot\vec{q}\left[p^i\delta_{jk}\mathcal{P}_{1\gamma} + p^j\delta_{ik}\mathcal{P}_{2\gamma}\right] + p^ip^jq^k\mathcal{P}_{3\gamma} + \cdots, \tag{8}
$$

where $\mathcal{P}_{i\gamma}(i = 1,2,3)$ are Lorentz form factors depending on $m_{p\bar{p}}$.

The charge conjugation gives a constraint that $\vec{F}_{\gamma\sigma}$ must *exactly* go to zero when $|\vec{q}|\to 0$, thus, $\vec{F}_{\gamma\sigma}$ is not singular as $|\vec{q}|\to 0$. It is well-known that Coulomb interaction can cause a singularity in the amplitude when $|\vec{q}|\to 0$. From our above discussion one sees that the singularity can only appear in $F_{\gamma 0}$, i.e., in the contribution with $S = 0$.

We now discuss the enhancement effect near the threshold observed at BES[1] using results obtained in the above. In [1] the effect is interpreted as an existence of a resonant structure near the threshold. The resonant structure can be an S-wave state or a P-wave state. If it is an S-wave state, it must be the state with the quantum numbers ${}^1S_0$, or $J^{PC} = 0^{-+}$. If it is a P-wave state, it must be the state with the quantum numbers ${}^3P_J$, or $J^{PC} = J^{++}$ with $J = 0, 1, 2$. The effect of the resonant structure manifest itself via $F_{\gamma 0}$, or $\vec{F}_{\sigma\gamma}$. Beside possible enhancement effects due to the resonant structure near the threshold, some other enhancement mechanisms should be in these form factors for $|\vec{q}| \to 0$ in order to compensate the suppression factor $|\vec{q}| \to 0$ from the phase-space near the threshold. Coulomb interaction may do the job[8].

In a similar way we can write the form factors for $J/\psi(\vec{p}, \vec{\varepsilon}_J) \to \pi^0(\vec{p}) + p(\vec{q}) + \bar{p}(-\vec{q})$. The amplitude which respects P and C symmetries can be written as

$$
\begin{aligned}
&\mathcal{T}(J/\psi \to \pi^0 p\bar{p}) = \psi^\dagger \left(F_{\pi 0}(\vec{p}, \vec{q}) + \vec{\sigma} \cdot \vec{F}_{\pi\sigma}(\vec{p}, \vec{q}) \right) \chi \\
&= \psi^\dagger \Big\{ (\vec{\varepsilon}_J \cdot \vec{q} \mathcal{P}_{1\pi} + \vec{\varepsilon}_J \cdot \vec{p}\vec{p} \cdot \vec{q} \mathcal{P}_{2\pi}) + (\vec{\sigma} \times \vec{\varepsilon}_J) \cdot \vec{p} \mathcal{S}_\pi \\
&+ \Big(q^i q^j - \frac{1}{3} |\vec{q}|^2 \delta_{ij} \Big) \cdot \Big[(\vec{\sigma} \times \vec{\varepsilon}_J) \cdot \vec{p} p^i p^j \mathcal{D}_{1\pi} \\
&+ (\vec{\sigma} \times \vec{\varepsilon}_J)^i p^j \mathcal{D}_{2\pi} + (\vec{\sigma} \cdot \vec{p} (\vec{\varepsilon}_J \times \vec{p})^i p^j \\
&+ \vec{\varepsilon}_J \cdot \vec{p} (\vec{\sigma} \times \vec{p})^i p^j \mathcal{D}_{3\pi} + (\sigma^i (\vec{\varepsilon} \times \vec{p})^j \\
&+ \varepsilon_J^i (\vec{\sigma} \times \vec{p})^j) \mathcal{D}_{4\pi} \Big] \Big\} \chi + \cdots,
\end{aligned} \tag{9}
$$

where we have neglected contributions from states with $L > 2$. From the above a possible resonant structure near the threshold for this decay could only be the state with $S = 0$ and $L = 1$ and the quantum numbers $J^{PC} = 1^{+-}$, or with $S = 1$ and $L = 0, 2$ and the quantum numbers $J^{PC} = J^{--}$ where J can be 1,2,3. Since the quantum numbers here are different from those of the resonant structure in $J/\psi \to \gamma p\bar{p}$, the observation of the resonant structure in $J/\psi \to \gamma p\bar{p}$ does not necessarily indicate the existence of a resonant structure in $J/\psi \to \pi^0 p\bar{p}$.

Now we turn to the observed enhancement near the threshold in $J/\psi \to K^+ \Lambda \bar{p}$[2]. One needs to understand why there is no enhancement in $\pi^0 p\bar{p}$ whereas there is one in $K^+ \Lambda \bar{p}$. Because the final state $K^+ \Lambda \bar{p}$ is not an eigenstate of the charge conjugation, we cannot use the symmetry of charge conjugation to constrain the decay amplitude in the same way as above. However, under $SU(3)$ the two decays are related since the mesons π^0, K^+ and the

baryons p and Λ belong to octets of $SU(3)$. Therefore combining $SU(3)$ with the C and P symmetries, one can obtain additional constraints. This combined CPS[12] symmetry has been shown to be extremely powerful in the lattice calculations of hadronic matrix elements. One can expect that the CPS symmetry will set further constraints on $J/\psi \to \pi^0 p\bar{p}$ and $J/\psi \to K^+\Lambda\bar{p}$ decays and their threshold enhancements.

The J/ψ transforms as an $SU(3)$ singlet whereas π^0, K^+, p and Λ transform as components of an $SU(3)$ octet. Using flavor $SU(3)$ symmetry one can write down relations between different decay amplitudes for J/ψ decays into a pseudoscalar in the octet M : $(\pi^{\pm,0}, K^{\pm}, \bar{K}^0, K^0, \eta)$ plus a baryon and an anti-baryon in the octet $B : (\Sigma^{-,0,+}, p, n, \Xi^{-,0}, \Lambda)$. The decay amplitudes respecting the $SU(3)$ symmetry depend on two types of amplitudes $\tilde{F}$ and $\tilde{D}$ similar to pseudoscalar nucleon couplings. Again we will use two-component fields for baryons, hence the matrix elements in B are two component spinors. We can also decompose the $SU(3)$ fields as: $B = \sqrt{2}\psi_B^{a\dagger}T^a$, $\bar{B} = \sqrt{2}\chi_{\bar{B}}^a T^a$, $M = \sqrt{2}M^a T^a$. Here T^a is the $SU(3)$ generators normalized as $Tr(T^aT^b) = \delta^{ab}/2$. The amplitude with the $SU(3)$ symmetry for $J/\psi(\vec{p}, \vec{\varepsilon}_J) \to M(\vec{p}) + B(\vec{q}) + \bar{B}(-\vec{q})$ can be written as:

$$
\begin{aligned}
&\mathcal{T}(J/\psi \to MB\bar{B}) \\
&= \mathrm{Tr}\left[(\tilde{D}(\vec{\varepsilon}_J, \vec{p}, \vec{q})\{M, B\} + \tilde{F}(\vec{\varepsilon}_J, \vec{p}, \vec{q})[M, B])\bar{B}\right] \\
&= \sqrt{2}M^b\psi_B^{a\dagger}\left\{d_{abc}\tilde{D}(\vec{\varepsilon}_J, \vec{p}, \vec{q}) + if_{abc}\tilde{F}(\vec{\varepsilon}_J, \vec{p}, \vec{q})\right\}\chi_{\bar{B}}^c,
\end{aligned}
\tag{10}
$$

where f_{abc} and d_{abc} are the $SU(3)$ structure constants. $\tilde{F}$ and $\tilde{D}$ are 2×2 matrices acting in the spin space. Under charge conjugation the bilinear products of baryon fields and the meson fields transforms as: $\psi_B^{a\dagger}\chi_{\bar{B}}^b \to c_b c_a \psi_B^{b\dagger}\chi_{\bar{B}}^a$, $\psi_B^{a\dagger}\vec{\sigma}\chi_{\bar{B}}^b \to -c_a c_b \psi_B^{b\dagger}\vec{\sigma}\chi_{\bar{B}}^a$, $M^a \to c_a M^a$, where $c_a = 1$, for $a = 1, 3, 4, 6, 8$, $c_a = -1$, for $a = 2, 5, 7$. For the $SU(3)$ structure constants f_{abc} and d_{abc}, we have: $f_{abc}c_a c_b c_c = -f_{abc}$, $d_{abc}c_a c_b c_c = d_{abc}$. Parity transformation is as usual.

Neglecting here all contributions with $L > 1$, $\tilde{F}$ and $\tilde{D}$ can be parameterized as:

$$
\begin{aligned}
\tilde{F} &= \vec{\varepsilon}_J \cdot \vec{q}\mathcal{P}_{1F} + \vec{\varepsilon}_J \cdot \vec{p}\vec{p} \cdot \vec{q}\mathcal{P}_{2F} + (\vec{\sigma} \times \vec{\varepsilon}_J) \cdot \vec{p}\mathcal{S}_F + \cdots \\
\tilde{D} &= \vec{\varepsilon}_J \cdot \vec{q}\mathcal{P}_{1D} + \vec{\varepsilon}_J \cdot \vec{p}\vec{p} \cdot \vec{q}\mathcal{P}_{2D} + (\vec{\sigma} \times \vec{\varepsilon}_J) \cdot \vec{p}\mathcal{S}_D + \cdots,
\end{aligned}
\tag{11}
$$

where all form factors $\mathcal{P}_{(1,2)F}$, $\mathcal{P}_{(1,2)D}$ and $\mathcal{S}_{F,D}$ depend only on the invariant mass of $m_{B'\bar{B}}$.

The general feature of the decay amplitude is that if the state $B'\bar{B}$ is in the state with even L, it must be in a spin triplet with $J^P = J^-$. If the state $B'\bar{B}$ is in the state with odd L, it must be in a spin singlet with $J^P = L^+$. From the general amplitude in Eq.(10) we can find various decay amplitudes for different decays, which are listed in Table I. In Table I all symbols for baryons or anti-baryons stand for two- component spinors respectively.

TABLE I: $SU(3)$ amplitudes for $J/\psi \to MB'\bar{B}$.

π^0	$\frac{1}{\sqrt{2}}\bar{p}(\tilde{D}+\tilde{F})p - \frac{1}{\sqrt{2}}\bar{n}(\tilde{D}+\tilde{F})n$ $-\frac{1}{\sqrt{2}}\bar{\Xi}^0(\tilde{D}-\tilde{F})\Xi^0 + \frac{1}{\sqrt{2}}\bar{\Xi}^-(\tilde{D}-\tilde{F})\Xi^-$ $+\sqrt{2}(\bar{\Sigma}^+\tilde{F}\Sigma^+ - \bar{\Sigma}^-\tilde{F}\Sigma^-) + \sqrt{\frac{2}{3}}(\bar{\Sigma}^0\tilde{D}\Lambda + \bar{\Lambda}\tilde{D}\Sigma^0)$
π^+	$\bar{p}(\tilde{D}+\tilde{F})n + \bar{\Xi}^0(\tilde{D}-\tilde{F})\Xi^-$ $+\sqrt{2}(\bar{\Sigma}^0\tilde{F}\Sigma^- - \bar{\Sigma}^+\tilde{F}\Sigma^0) + \sqrt{\frac{2}{3}}(\bar{\Sigma}^+\tilde{D}\Lambda + \bar{\Lambda}\tilde{D}\Sigma^-)$
K^0	$\bar{p}(\tilde{D}-\tilde{F})\Sigma^+ - \frac{1}{\sqrt{2}}\bar{n}(\tilde{D}-\tilde{F})\Sigma^0$ $+\bar{\Sigma}^-(\tilde{D}+\tilde{F})\Xi^- - \frac{1}{\sqrt{2}}\bar{\Sigma}^0(\tilde{D}+\tilde{F})\Xi^0)$ $-\frac{1}{\sqrt{6}}\bar{n}(\tilde{D}+3\tilde{F})\Lambda - \frac{1}{\sqrt{6}}\bar{\Lambda}(\tilde{D}-3\tilde{F})\Xi^0$
K^+	$\bar{n}(\tilde{D}-\tilde{F})\Sigma^- + \frac{1}{\sqrt{2}}\bar{p}(\tilde{D}-\tilde{F})\Sigma^0$ $+\bar{\Sigma}^+(\tilde{D}+\tilde{F})\Xi^0 + \frac{1}{\sqrt{2}}\bar{\Sigma}^0(\tilde{D}+\tilde{F})\Xi^-$ $-\frac{1}{\sqrt{6}}\bar{p}(\tilde{D}+3\tilde{F})\Lambda - \frac{1}{\sqrt{6}}\bar{\Lambda}(\tilde{D}-3\tilde{F})\Xi^-$
η	$-\frac{1}{\sqrt{6}}\bar{p}(\tilde{D}-3\tilde{F})p - \frac{1}{\sqrt{6}}\bar{n}(\tilde{D}-3\tilde{F})n$ $-\frac{1}{\sqrt{6}}\bar{\Xi}^0(\tilde{D}+3\tilde{F})\Xi^0 - \frac{1}{\sqrt{6}}\bar{\Xi}^-(\tilde{D}+3\tilde{F})\Xi^-$ $+\sqrt{\frac{2}{3}}(\bar{\Sigma}^-\tilde{D}\Sigma^- + \bar{\Sigma}^0\tilde{D}\Sigma^0 + \bar{\Sigma}^+\tilde{D}\Sigma^+) - \sqrt{\frac{2}{3}}\bar{\Lambda}\tilde{D}\Lambda$

Since there is not an enhancement in $\pi^0 p\bar{p}$ whose amplitude is proportional to $\tilde{F}+\tilde{D}$, but there is an enhancement in $K^+\Lambda\bar{p}$ whose amplitude is proportional to $3\tilde{F}+\tilde{D}$, there should be a cancellation between $\tilde{F}$ and $\tilde{D}$ near threshold which results in $\tilde{F}+\tilde{D}\approx 0$ and $\tilde{F}\neq 0$. With this cancellation we have a consistent explanation for observation and non-observation of threshold enhancement in $J/\psi \to K^+\Lambda\bar{p}$ and $J/\psi \to \pi^0 p\bar{p}$, respectively. It is clear that any other process whose amplitude is not proportional to $\tilde{F}+\tilde{D}$ should show an enhancement near threshold. In the $SU(3)$ limit, the relative strength of enhancements near threshold in different channels are fixed if one only keeps S-wave contributions for small $|\vec{q}|$.

To show the relative enhancement, we define the ratio:

$$R(MB'\bar{B}) = \frac{|\mathcal{T}(J/\psi \to MB'\bar{B})|^2}{|\mathcal{T}(J/\psi \to K^+\Lambda\bar{p})|^2}, \tag{12}$$

where spin summations are implied in the amplitude squared. In experiment the amplitude squared can be measured, hence the defined ratios. In the $SU(3)$ limit, the ratios $R(MB'\bar{B})$ can be easily read off from Table I. For example for decays involving a K^+, we have: $R(K^+\Sigma^-\bar{n}) : R(K^+\Sigma^0\bar{p}) : R(K^+\Lambda\bar{p}) : R(K^+\Xi^-\bar{\Lambda}) = 6 : 3 : 1 : 4$, for a given $|\vec{q}|^2$.

In a realistic situation, there are $SU(3)$ breaking effects. For example, a source of the breaking effects comes from the splitting in meson masses and baryon masses. The enhancement in some channels may be weakened by $SU(3)$ breaking effects. Some of them are even forbidden kinematically, such as $J/\psi \to \eta\Xi^-\bar{\Xi}^-(\Xi^0\bar{\Xi}^0)$ although the corresponding matrix elements are not zero. Part of $SU(3)$ breaking effects due to mass splitting comes from $\vec{p}$ in the amplitude in Eq.(11). To have some ideas about the mass splitting effect on the threshold enhancements we neglect contributions from P-wave or higher. Then the matrix element squared is proportional to $|\vec{p}|^2$. $|\vec{p}|^2$ varies in different channels because of the mass differences. This factor will modify the ratio. For example:

$$\begin{aligned} &R(K^0\Sigma^+\bar{p}) \\ &= \frac{m^2_{\Lambda\bar{p}}}{m^2_{\Sigma^+\bar{p}}} \frac{(M^2_{J/\psi} - m^2_{\Sigma^+\bar{p}} - m^2_{K^0})^2 - 4m^2_{\Sigma^+\bar{p}}m^2_{K^0}}{(M^2_{J/\psi} - m^2_{\Lambda\bar{p}} - m^2_{K^+})^2 - 4m^2_{\Lambda\bar{p}}m^2_{K^+}} \times 6. \end{aligned}$$

One should take this ratio as a function of $|\vec{q}|^2$ to compare with experimental data for small $|\vec{q}|^2$. In this case $m_{\Lambda\bar{p}}$ and $m_{\Sigma^+\bar{p}}$ are the threshold masses $m_\Lambda + m_p$ and $m_{\Sigma^+} + m_p$. Similarly one can obtain the ratios for other decays. We obtain the non-zero ratios with: $R(\pi^0\Xi^0\bar{\Xi}^0) : R(\pi^0\Xi^-\bar{\Xi}^-) : R(\pi^0\Sigma^+\bar{\Sigma}^+) : R(\pi^0\Sigma^-\bar{\Sigma}^-) : R(\pi^0\Lambda\bar{\Sigma}^0) : R(\pi^0\Sigma^0\bar{\Lambda}) = 0.54 : 0.51 : 1.50 : 1.42 : 0.63 : 0.63$, $R(\pi^+\Xi^0\bar{\Xi}^-) : R(\pi^+\Sigma^0\bar{\Sigma}^+) : R(\pi^+\Sigma^-\bar{\Sigma}^0) : R(\pi^+\Lambda\bar{\Sigma}^+) : R(\pi^+\Sigma^-\bar{\Lambda}) = 1.04 : 1.48 : 1.44 : 0.63 : 0.62$, $R(K^0\Sigma^+\bar{p}) : R(K^0\Sigma^0\bar{n}) : R(K^0\Lambda\bar{n}) : R(K^0\Xi^0\bar{\Lambda}) = 4.71 : 2.32 : 0.99 : 0.77$, $R(K^+\Sigma^-\bar{n}) : R(K^+\Sigma^0\bar{p}) : R(K^+\Lambda\bar{p}) : R(K^+\Xi^-\bar{\Lambda}) = 4.59 : 2.35 : 1. : 0.75$, $R(\eta p\bar{p}) : R(\eta n\bar{n}) : R(\eta\Sigma^+\bar{\Sigma}^+) : R(\eta\Sigma^0\bar{\Sigma}^0) : R(\eta\Sigma^-\bar{\Sigma}^-) : R(\eta\Lambda\bar{\Lambda}) = 6.28 : 6.24 : 0.22 : 0.21 : 0.19 : 0.48$.

From the above analysis one sees that there are many channels where a near threshold enhancement can happen and in some of the channels the enhancement is even larger than the enhancement in $J/\psi \to K^+\Lambda\bar{p}$. A systematic search for near threshold enhancement in all channels listed in the above can reveal the detailed dynamics inducing the enhancement.

One can carry out a similar $SU(3)$ analysis for $J/\psi \to \gamma B'\bar{B}$ decays by parameterizing the amplitudes as

$$
\begin{aligned}
\mathcal{T} &= Tr[\bar{B}(D_\gamma(\varepsilon_J, \varepsilon_\gamma, p, q)\{Q, B\} \\
&\quad +F_\gamma(\varepsilon_J, \varepsilon_\gamma, p, q)[Q, B])] \\
&= \bar{p}(F_\gamma + \frac{1}{3}D_\gamma)p + \bar{\Sigma}^+(F_\gamma + \frac{1}{3}D_\gamma)\Sigma^+ \\
&- \bar{\Sigma}^-(F_\gamma - \frac{1}{3}D_\gamma)\Sigma^- - \bar{\Xi}^-(F_\gamma - \frac{1}{3}D_\gamma)\Xi^- \\
&- \frac{2}{3}(\bar{n}D_\gamma n + \bar{\Xi}^0 D_\gamma \Xi^0) - \frac{1}{3}\bar{\Lambda}D_\gamma\Lambda \\
&+ \frac{1}{3}\bar{\Sigma}^0 D_\gamma \Sigma^0 + \frac{1}{\sqrt{3}}(\bar{\Lambda}D_\gamma\Sigma^0 + \bar{\Sigma}^0 D_\gamma \Lambda).
\end{aligned} \tag{13}
$$

The general forms of F_γ and D_γ are given in eqs. (6) and (8), respectively.

There may be cancellations among the F_γ and D_γ terms in certain channels, but not all of them. One therefore also expects to observe enhancements in several other channels, if the enhancement in $J/\psi \to \gamma p\bar{p}$ is confirmed. Since two amplitudes are needed to specify the complete amplitudes, one needs to measure another channel to fix the parameters.

In this work we have not dealt with the dynamical mechanism for the enhancement in $J/\psi \to \gamma p\bar{p}$ and $J/\psi \to K^+\Lambda\bar{p}$. At present there is not enough information to decide the detailed mechanism. We emphasize, however, that the results obtained in this paper are independent of any detailed dynamics for the enhancement effects. Our predictions based on symmetry principles are therefore crucial in further confirming the enhancement effects. Detailed study of predictions in different channels will surely provide important information about the enhancement effects. We strongly urge our experimental colleagues to carry out systematic analysis about threshold enhancement in $J/\psi \to \gamma B'\bar{B}$ and $J/\psi \to MB'\bar{B}$.

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